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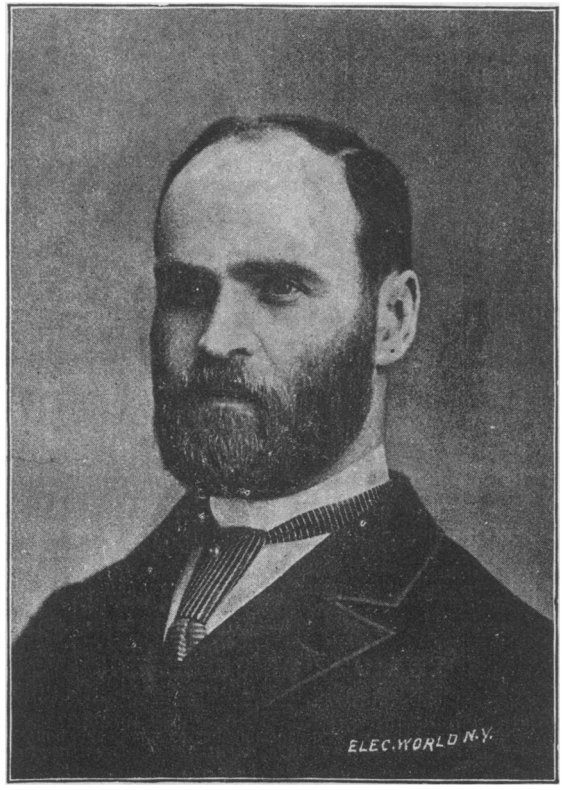
BIOGRAPHY.

ALEXANDER MACFARLANE, M. A., D. Sc., LL. D.

BY J. M. COLAW.

ALEXANDER MACFARLANE was born at Blairgowrie, Scotland, April 21st, 1851. He was educated at the public school, and at 13 became a regular pupil teacher in the employment of the Education Department.

In 1869, having finished his apprenticeship as a teacher and saved a little money, Mr. Macfarlane went straight to the University of Edinburgh. At that time the curriculum for Master of Arts consisted of three departments, classical, mathematical, and philosophical; and it was customary for the more ambitious students to take the degree with honors in one of these departments. Mr. Macfarlane first entered the Junior classes in Latin and in Greek, and at the end of the session stood fourth in the former and fifth in the latter, in classes of 200, largely composed of High School graduates. He perceived that to carry himself through college it was necessary either to sacrifice a large part of his time to teaching, or else to study hard and pay his way by means of money prizes. He chose the bolder alternative. At the beginning of his second year he won in open competition the Miller scholarship, worth $400. At the end of that year he stood very high in Senior Latin and Greek and in Junior Mathematics. At the beginning of the third year he won in open competition the Spence scholarship, worth $1,000. The financial difficulty was now solved; there remained a choice of a department for honors. He was urged by the professor of Latin to go forward in the Classics, but he felt that there was more scope for originality in philosophy. In his third year he studied Senior Mathematics, Natural Philosophy and Logic. It was the custom of Professor Kelland to introduce **Quaternions to his senior students.** The addition of vectors



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was intelligible, but the product of vectors seemed to be a universal difficulty. The professor explained that in $i\,j$ the left-hand vector was to be considered as a sort of corkscrew turning the right-hand vector through a right angle; but he did not explain how in $i\,i$ it ceased to be a corkscrew. To get light on the subject Mr. Macfarlane bought a copy of Tait's *Treatise on Quaternions*, but found that it was addressed to mathematicians.

Before he entered the class of Logic Mr. Macfarlane was familiar with the works of Hamilton and Mill, and when a member of the class he read, at the invitation of the professor, a paper which criticised the statement of the law of Excluded Middle given by Jevons in his *Lessons on Logic*. It was his intention to study for honors in Logic and Philosophy, but perceiving how much they depended on the principles of science, and especially of exact science, he took up the advanced classes in Mathematics and Physics as a secondary study. In Experimental and Mathematical Physics he gained the highest honors and the personal friendship of Professor Tait, then, as now, the greatest figure in the University. In 1874 he was appointed Neil Arnott instructor in Physics, and in 1875 finished an unusually extensive course of undergraduate study by taking the degree of M. A. with honors in Mathematics and Physics. The University record showed that he had passed each of the seven subjects of the pass examinations with high distinction. Having, after graduation, won in a competitive examination the Maclaren fellowship, worth $1,500, he proceeded to study for the recently instituted degree of Doctor of Science. After one year spent on Chemistry, Botany, and Natural History, and two years on Mathematics and Physics, he obtained the doctorate in 1878. His thesis was an experimental research on the conditions governing the electric spark, and it was subsequently published in the *Transactions of the Royal Society of Edinburgh*. It also brought him under the notice of the celebrated electrician and philosopher, Clark Maxwell, who made various suggestions for its extension.

In 1878 Dr. Macfarlane was elected a Fellow of the Royal Society of Edinburgh, and the first contribution which he read personally was a memoir on the Algebra of Logic. The memoir was referred by the Council to the professors of mathematics and of logic, and they reported that it was too mathematical for the one and too logical for the other to enable them to say what its value was. Dr Macfarlane enlarged the memoir and published it as a small volume under the title of *Principles of the Algebra of Logic* (1879). The volume was received with favor, and brought the author into correspondence with Munro, Jevons, Venn, Cayley, Harley, Schroeder and Halsted, who was then lecturing on the mathematical logicians at Johns Hopkins University. The main idea propounded is that of a limited and definite universe; also Euler's diagrams were further developed. In 1879 he attended the meeting of the British Association at Sheffield, and there met many of the British savants.

During 1880 Dr. Macfarlane was interim Professor of Physics at the University of St. Andrews, and in 1881 he was appointed for the usual period of three years Examiner in Mathematics in the University of Edinburgh. During these years he contributed to the Royal Society of Edinburgh a series of

experimental papers on electricity, and a series of mathematical papers on the *Analysis of the Relationships of Consanguinuity and Affinity*. A paper on this subject, which he read before the Anthropological Institute of London, contains as perfect a notation for relationship as is the Arabic notation for numbers. These papers, as well as those on the Algebra of Logic, now form part of the history of Exact Logic. He also contributed to the Royal Society of Edinburgh a *Note on Plane Algebra*, which stated briefly the view he had arrived at concerning the imaginary algebra of the plane. It states that the fundamental quantity is versor rather than a vector, a view in advance of Argand's, and indeed of much that has been written more recently. By means of this algebra of the plane he deduced many series, some of which he propounded as problems in the *Educational Times* and the *Mathematical Visitor*. It was also during his tenure of office as examiner that he prepared the volume on *Physical Arithmetic*, a pioneer work, whose express object is to elucidate the logical processes involved in the application of arithmetic to physical problems.

In 1885 Dr. Macfarlane was called to the chair of physics at the University of Texas, where he became a colleague of his fellow logician, Dr. Halsted. That same year he met many of the American savants at the Ann Arbor meeting of the American Association. In 1887 he received the honorary degree of LL. D. from the University of Michigan on the occasion of their semi-centennial. His first years at the University of Texas were wholly taken up with organizing the department, but in 1889 he published as a sequel to *Physical Arithmetic* a volume of *Elementary Mathematical Tables*, distinguished for their comprehensiveness and uniformity. In 1889 he visited Paris at the time of the Exposition and met many of the continental savants at the meeting of the French Association.

On his return from Europe, he began to publish the results of his study of the algebra of space, which he approached as a logical generalization of the Algebra of the Plane. These papers are as follows: 1°. *Principles of the Algebra of Physics*, read before the Washington meeting of the American Association in 1891, states the fundamental difficulties in the theory of Quaternions, lays stress on the distinction between vectors and versors, and deals mostly with the products of vectors. 2°. *On the Imaginary of Algebra*, read at the Rochester meeting in 1892, gives an historical and critical account of the different interpretations of $\surd(-1)$, takes up the functions of versors, and shows that there are at least two distinct geometrical meanings of $\surd(-1)$. 3°. *The Fundamental Theorems of Analysis Generalized for Space*, contributed to the New York Mathematical Society in 1892, investigates and proves the generalized form of the Binomial and other theorems, and thus establishes the prineiples of spherical trigonometrical analysis. 4°. *On the Definitions of the Trigonometric Functions*, read before the Mathematical Congress at Chicago in 1893, defines these functions so as to apply to the circle, hyperbole, ellipse, logarithmic spiral, and a complex curve partly circular, partly hyperbolic. 5°. *The Principles of Elliptic and Hyperbolic Analysis*, read at the same place and time, extends spherical trigometrical analysis to the other surface of the second

order. 6°. *The analytical treatment of alternating currents*, read before the International Electrical Congress at the same time, shows that plane algebra is the analysis needed for the problems of alternating currents 7°. *On physical addition or composition*, read before the Madison meeting of the American Association in 1893, treats in a uniform manner of the composition of various physical quantities located in space, ending with the composition of screw-motions. 8°. *On the fundamental principles of exact analysis*, read before the Philosophical Society of Washington in 1894, discusses the fundamental laws of algebra, and the logical principle of generalization in analysis. 9°. *The principles of differentiation in space analysis*, recently read before the American Mathematical Society at New York, investigates the differentiation of versors, and publishes the true generalization of Taylor's theorem for space.

In 1891 Dr. Macfarlane took an active part in organizing the Texas Academy of Science, and for two years acted as its Honorary Secretary. He contributed many papers, among which may be mentioned "An Account of the Rainmaking Experiments in San Antonio," an article describing and criticising the various modern methods of rainmaking, and a paper on "Exact Analysis as the Basis of Language," where his knowledge both of languages and of mathematics comes into play.

In 1894 Professor Macfarlane resigned from the University of Texas. Throughout the nine years he labored there, he gave the new University the full benefit of his varied experience as a teacher, his accurate knowledge of University affairs, and his widespread reputation as a savant. The course in mathematical physics was so well developed as to call forth a special article in the *Rievista di Matematica*, published at Turin, Italy.

Professor Macfarlane, in addition to being a member of numerous American and British societies, is a corresponding member of the *Sociedad Cientifica Antonio Alzate*, of Mexico, and the *Circolo Matematico di Palermo*, Italy. Personally he is a characteristic Scotsman, sturdy, persevering, with a relish for hard work, thoughtful, courageous in his convictions, and endowed with more than the average share of the *perfervidum ingenium Scotorum*. He is unmarried, but it is announced that in this, as in other matters, good fortune awaits him. And as he is still a young man, it is not likely that we have seen the last of his contributions to mathematical analysis.

To the editors of the *Electrical World* we are indebted for the loan of the electrotype.